Technical Notes

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Shock Tube Flow Passing Through a Section of a Linear MHD Generator

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Introduction

THE interaction of a shock tube flow with a section of linear MHD generator has been discussed by many authors.[1-4] In the first paper considering an outside circuit,[1] the assumption of a semi-infinite generator section and semi-infinite shock tube flow resulted in a large shock wave damping. An experiment by Zauderer and Tate[4] of shock tube flow with a relatively short MHD section and short test time showed small over-all change of shock wave velocity. Using the modified two-step Lax-Wendroff finite difference scheme, the interactions of the shock wave with a finite section of a linear MHD generator were calculated.[2] These calculations were compared with experimental results.[4] They show that inclusion of ionization energy in the internal energy term is very important, and that the assumption of constant average $\gamma = c_p/c_v$ leads to too high a degree of ionization and higher temperatures than those indicated by the experiment. However, calculations taking into account ionization energy were performed in Ref. 2 only in the case of a semi-infinite slug of gas. The contact surface was included only in the case of constant γ. The shock wave was smeared over a few mesh points which is typical for the Lax-Wendroff technique. This made it difficult to introduce the ionization relaxation observed experimentally. In the present paper the system of coordinates was taken to move with the shock wave, which enabled the treatment of the primary shock wave as a sharp discontinuity rather than being smeared over a few meshes. This in turn allowed the introduction of the ionization relaxation time, taken from the experiment. A slug of the gas was assumed to be finite and equal to the corresponding value of the experimental results.[4] Propagation of a contact surface (turning off electric conductivity) through an MHD zone was treated by using the iteration procedure of fitting conditions across the contact surface and extending the hot flow (pseudoflow) for numerical convenience by one mesh point to the other side of the contact surface. In order to approach experimental conditions in xenon gas as closely as possible, the electric field distribution, rather than being calculated from the given outside load, was taken directly from Zauderer and Tate's measurement[4] (corresponding to a load of 0.23 ohm) with additional correction for electrode loss. Ionization was assumed to be in equilibrium with gas temperature equal to that of electrons. The magnetic Reynolds number was assumed to be small and the induction field was neglected.

System of Equations and Numerical Technique

Introducing the coordinate transformation to the system moving with the shock wave of velocity $U = U(t)$

$$\tau = t, \quad \xi = \int_0^t U(t)\,dt - x \qquad \bar{u} = U - u \tag{1}$$

the system of equations[2] can be transformed into the following form:

$$\frac{\partial \bar{W}}{\partial \tau} = \frac{\partial \bar{F}}{\partial \xi} + \bar{S} \tag{2}$$

with

$$\bar{W} = \begin{pmatrix} \rho \\ \rho\bar{u} \\ \rho\left(\frac{\bar{u}^2}{2} + e + \frac{V_i}{m}\alpha\right) \end{pmatrix} \qquad \bar{F} = -\begin{pmatrix} \rho\bar{u} \\ \rho\bar{u}^2 + p \\ \rho\bar{u}\left(\frac{\bar{u}^2}{2} + h + \frac{V_i}{m}\alpha\right) \end{pmatrix}$$

$$\bar{S} = \begin{pmatrix} 0 \\ \rho\dfrac{dU}{d\tau} + j_y B_z \\ \rho\bar{u}\dfrac{dU}{d\tau} - Q + j_y(B_z U + E_y) \end{pmatrix}$$

where the standard notations have been used as in Ref. 2.

The electric current density is calculated from Ohm's law

$$j_y = -\sigma(E_y - uB_z) = -\sigma(E_y + \bar{u}B_z - UB_z) \tag{3}$$

The radiation loss has been calculated from the empirical equation[4] (p. 501). The equation of state reads

$$p = (1+\alpha)(k/m)T \tag{4}$$

where k is the Boltzmann constant and m, the atomic mass.

The electric conductivity can be calculated from the Lin-Kantrowitz formula

$$\sigma = \left(\frac{2.6\times 10^{-6}T^{1/2}}{Q_{en}} + \frac{65.5\ln\Lambda}{T^{3/2}}\right)^{-1} \tag{5}$$

Q_{en} denotes the electron-neutral cross section (m^2). Λ denotes Spitzer's Λ. Electric conductivity was assumed to be switched on behind the shock wave at the distance

$$\xi_r = \int_0^{t_r} U\,dt$$

where relaxation time t_r was taken from the experimental observation[4] equal to 15 μsec. The degree of ionization α has been calculated from Saha's equation.

The numerical procedure is similar to that of Ref. 2, except for the fact that due to transformation of coordinates, the source term S contains an unknown derivative of the shock wave velocity $dU/d\tau$. The iteration procedure around the shock was the same as described earlier.[5]

The iteration procedure around the interface is based on the assumption that at the cold side of the contact surface the simple wave relation is satisfied. (In the nonconducting gas there is no interaction with the magnetic field or energy exchange.)

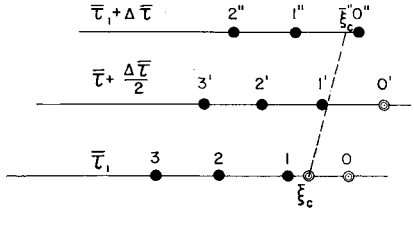

Fig. 1 Numerical procedure around the contact surface.

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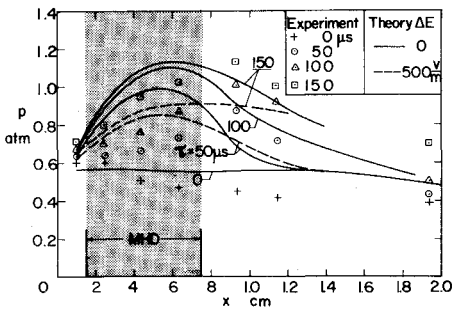

Fig. 2 Comparison of pressure distribution with experimental data with and without electrode loss (τ is time which elapsed from the passage of the shock wave at the given position).

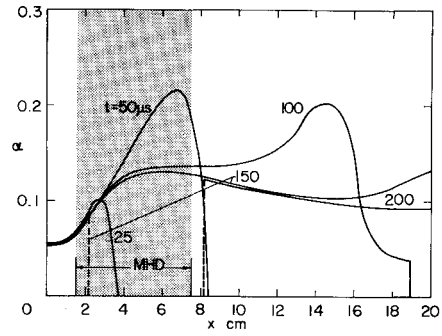

Fig. 4 Distribution of degree of ionization in laboratory system of coordinates.

This assumption could be violated if the secondary shock wave propagates upstream crossing the contact surface. However, even then, for a relatively weak shock wave the following simple wave relation is approximately valid:

$$p_c/p_4 = \{1-[\gamma_4-1)/2]u_c/a_4\}^{2\gamma_4/(\gamma_4-1)} \qquad (6)$$

where the subscript 4 denotes stagnation conditions of the driver gas. The numerical procedure at the contact surface $\xi = \xi_c$ is as follows. For a certain $\bar{\tau} = \bar{\tau}_1$ or initially ($\bar{\tau}_1 = 0$) (Fig. 1), all variables are known. After the correct value of $dU/d\bar{\tau}$ has been obtained from the iterative scheme at the shock front, all values at half-points $(\bar{\tau}_1+\Delta\bar{\tau}/2)$ except point 0′ and the points at $\bar{\tau}_1+\Delta\bar{\tau}$ can be calculated by using the basic Lax-Wendroff method. The pseudovalues at points 0 and 0′ were evaluated by extrapolation of the distribution at the left side of the contact surface. Values of the velocity and pressure in the point 0″ were calculated from known values in half-step points 1′ and 0′.

The position of the contact surface at $\bar{\tau}_1+\Delta\bar{\tau}$, $\bar{\xi}_c''$ was obtained by the following schemes.

1) If the velocities at points 1″ and 0″ were nearly the same, $\bar{u}_c'' = \frac{1}{2}[\bar{u}(1'')+\bar{u}(0'')]$ was assumed.

2) If the pressure at points 1″ and 0″ was nearly the same, $p_c'' = \frac{1}{2}[p(1'')+p(0'')]$ was assumed.

These two cases require no iteration. The corresponding pressure p_c or velocity $\bar{u}_c$ was obtained from the isentropic relation at the other side of the contact surface [Eq. (6)].

3) If conditions 1 or 2 are not satisfied, an iterative scheme must be applied.

Assuming $\bar{u}_c''$, we have $\bar{\xi}_c'' = \frac{1}{2}(\bar{u}_c+\bar{u}_c'')\Delta\bar{\tau}+\bar{\xi}_c$. Based on $\bar{\xi}_c''$, the $\bar{p}_c$ can be evaluated by extrapolation using a second-order (or higher) polynomial. The assumed $\bar{u}_c''$ was compared with the $\bar{u}_c''$ calculated from the isentropic relation (6). The iteration is to be repeated until the error is contained within a certain tolerance, or the scheme becomes divergent.

4) This case has been used for diverging scheme (3). By assuming $\bar{u}_c''$ or $\bar{\xi}_c''$, one gets a new value of $\bar{u}_c''$ by extrapolation. If this value is not in agreement with the assumed one, a new value is assumed until the correct $\bar{u}''$ is obtained. Equation (6) was used to find p_c''.

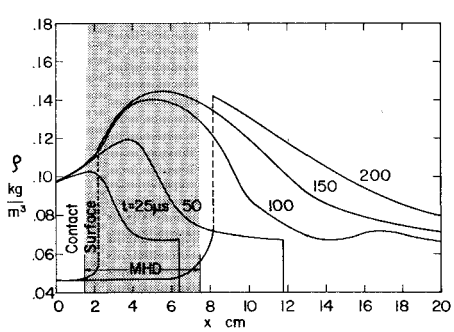

Fig. 3 Density distribution in laboratory system of coordinates.

After the correct values of p_c'', $\bar{u}_c''$, and $\bar{\xi}_c''$ are obtained, all the other variables at $\bar{\xi}_c''$ can be evaluated by extrapolation.

Numerical Results and Discussion

Sample numerical results are presented in Figs. 2–4. Figure 2 presents a comparison of the calculated pressure distribution with experimental data,[4] for a fixed time interval τ after the shock wave passes indicating a somewhat excessively steep theoretical pressure rise but otherwise a reasonable agreement. The initial discrepancy could be a result of induction effects neglected in the theoretical model. Inclusion of electrode loss (broken curve) of about 25 v corresponding to $\Delta E = 500$ v/m shows a much smaller pressure rise. Such an electrode loss is typical for experimental conditions. One can notice in Figs. 2 and 3 the steepening pressure in time, indicating the formation of a secondary shock wave. However, as soon as the contact surface enters the MHD zone, interaction turns off and the shock wave quickly disappears (Fig. 3). Calculations indicate that the shock wave velocity drops when passing through the interaction zone, but later recovers to the original value. This is in agreement with experimental data showing only a small percent of over-all shock velocity change. However, velocity of the contact surface (in the laboratory system of coordinates) experiences a sharp drop by passing through the interaction zone. Temperature in the MHD zone reaches a maximum value of 11,500°K just behind the moving shock and drops later to about 10,000°K, which is within a few percent of the experimentally observed values. Calculations indicate that radiation energy loss reaches a maximum value equal to 13% of $\rho_0 U_{SI}^2$ and plays an important role, especially due to great temperature sensitivity of the degree of ionization. Density distributions in the laboratory system of coordinates (different from that of pressure distribution) are plotted in Fig. 3, indicating the passing of the shock wave as well as the contact surface through the interaction zone. A local dip in the density curve at 100 μsec is associated with a large increase in ionization as presented in Fig. 4.

References

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